The DPC-Hysteresis Model in Two-Dimensional Magnetostatic Finite Element Analysis

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Within this paper a scheme for incorporating hysteresis losses in the finite element analysis of low-frequency applications is described. The employed hysteresis model is the DPC-model which is a vectorized Preisach operator. The focus lies on the development of a computation procedure consisting of a proper potential formulation, implementation of the hysteresis model and finding a robust iteration scheme. The paper is limited to the two-dimensional case to circumvent adressing the problem of eddy current calculation. Simulation results for an exemplary machine configuration are presented and analyzed. The main contribution of the paper is the combination of the DPC-Hysteresis model with a finite element scheme and the application of the resulting method to the calculation of hysteresis loss in exemplary electromagnetic devices.

***Index Terms*—Finite element analysis, Magnetic fields, Magnetic hysteresis, Magnetic losses**

I. Introduction

In ELECTRIC machine design the need to accurately predetermine the iron losses has become ever increasing. This is a consequence of the high requirements concerning efficiency and construction space which electric machines have to face. Although being physically correlated, according to the loss separation principle iron losses can be divided in three categories: hysteresis losses, eddy current losses, and excess losses. The computation of each loss type poses different demands towards the employed numerical scheme. In case of electric machines eddy current losses require the treatment of laminated media, whereas the excess loss demands for models that incorporate the over-proportional increase of iron losses with increasing rate of change in magnetization.

The calculation of hysteresis loss in conjunction with finite element analysis (FEA) requires the usage of a hysteresis operator. Many of these have been proposed over time, one of the most widely used is the Preisach operator. While being a mapping between scalar-valued functions in its original form, there are various approaches to extend it to vector-valued inputs and outputs. The DPC-Hysteresis model is such a vectorized Preisach operator. It has been introduced by Della Torre, Pinzaglia and Cardelli in [1] and [2] and further developed and analyzed in consecutive publications (e.g. [3]). It allows for reproducing the losses associated with rotating magnetic fields in ferromagnetic material in accordance to experimental results. This makes it a promising choice for modeling hysteresis behavior of electrical steels in FEA.

A major problem the model poses is related to determination of the model parameters. This problem, in analogy to the scalar Preisach model called identification problem, is not subject of the presented work. The focus within this paper is laid on the application of the DPC-Hysteresis model in finite element analysis. The result is a stable and robust numerical scheme which allows the calculation of hysteresis losses.

II. Potential formulation and weak form

Within this paper, only two dimensional domains Ω are considered. The calculations are executed using the magnetic scalar potential Φ in conjunction with the impressed current vector potential $\boldsymbol{T}_0$ resulting in the formulation [4]

$$\nabla \cdot (\mu_{\mathrm{opt}} \nabla \Phi) = \nabla \cdot (\mu_{\mathrm{opt}} \boldsymbol{T}_0) + \nabla \cdot \boldsymbol{R} \ . \tag{1}$$

The corresponding weak form is depicted in (2), where v_k refers to the employed test functions (appropriate boundary conditions have to be added).

$$\int_\Omega \mu_{\mathrm{opt}} \nabla v_k \cdot \nabla \Phi \,\mathrm{d}\Omega = \int_\Omega \mu_{\mathrm{opt}} \nabla v_k \cdot \boldsymbol{T}_0 \,\mathrm{d}\Omega + \int_\Omega \nabla v_k \cdot \boldsymbol{R} \,\mathrm{d}\Omega \tag{2}$$

In (1) and (2) $\boldsymbol{R}$ is used to describe the hysteresis non-linearity according to the polarization method and μ_{opt} is the fixed point parameter (see [4] for details), which implies that

$$\mathscr{B}(\boldsymbol{H}) = \mu_{\mathrm{opt}} \boldsymbol{H} + \boldsymbol{R} \ . \tag{3}$$

The coupling between the potential formulation and the hysteresis operator is achieved by calculating $\boldsymbol{R}$ according to (3) while setting

$$\mathscr{B}(\boldsymbol{H}) = \mu_0 \boldsymbol{H} + \mathscr{H}(\boldsymbol{H}) \ . \tag{4}$$

The implementation of the hysteresis operator $\mathscr{H}(\boldsymbol{H})$ introduced in (4) will be specified in the next section.

III. Hysteresis model

The discrete representation of the hysteresis model needs to be interpolated in order to have a continuous expression for the magnetization. The latter is done following the idea presented in [5]. The model consists of a finite number of elementary hysteresis operators (hysterons) being distributed along N_{x} and N_{y} points along x- and y-axis and having N_{r} different radii. Then the discretized model can be written as:

$$\mathscr{H}^{\mathrm{d}}_{\mathrm{DPC}}(\boldsymbol{H}(t_i)) = M_{\mathrm{s}} \sum_{k=1}^{N_r} \sum_{l=1}^{N_{\mathrm{x}} N_{\mathrm{y}}} Q_{\boldsymbol{x},l,k}(\boldsymbol{H}(t_i), \boldsymbol{H}(t_{i-1})) \tilde{P}_{l,k} \tag{5}$$

In (5) M_{s} is the saturation magnetization, $Q_{\boldsymbol{x},l,k}$ is the state vector of the elementary hysteresis operators and $\tilde{P}_{l,k}$ is the value of the discretized Preisach distribution function. Therefore $\sum_k \sum_l \tilde{P}_{l,k} = 1$ must hold.

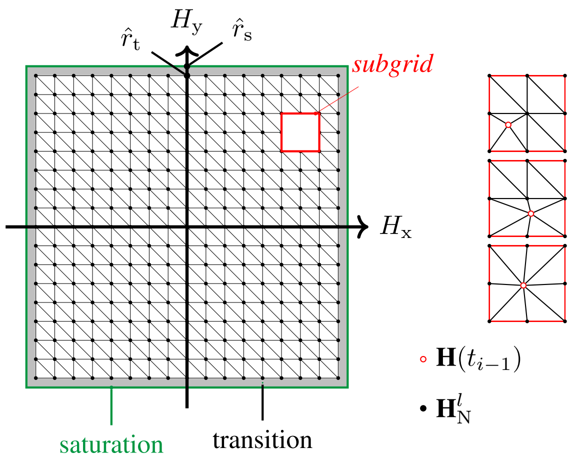

Fig. 1: Grid in the $\boldsymbol{H}$-plane of the continuous hysteresis model

In order to achieve an acceptable behavior in the applied fixed point iteration and to be able to construct the inverse model as well as apply the Newton-Raphson-algorithm (the latter are not treated within the paper), a continuous version of (5) is employed:

$$\mathscr{H}_{\mathrm{DPC}}^{\mathrm{d,lin}}(\boldsymbol{H}(t_i)) = \begin{cases} \boldsymbol{M}_{\mathrm{int}}(\boldsymbol{H}(t_i)), & \text{if} \quad ||\boldsymbol{H}(t_i)||_\infty - \hat{r}_{\mathrm{t}} < 0 \\ \boldsymbol{M}_{\mathrm{sat}}(\boldsymbol{H}(t_i)), & \text{if} \quad ||\boldsymbol{H}(t_i)||_\infty - \hat{r}_{\mathrm{s}} \geq 0 \\ \boldsymbol{M}_{\mathrm{tr}}(\boldsymbol{H}(t_i)), & \text{otherwhise} \end{cases} \tag{6}$$

The value of the magnetization $\boldsymbol{M}_{\mathrm{int}}$ is interpolated on a grid (see Fig. 1) according to the magnetization values of its neighboring nodes in the discretized $\boldsymbol{H}$-plane (see (7)). The symbol $\boldsymbol{M}_{\mathrm{sat}}$ refers to the saturation value and $\boldsymbol{M}_{\mathrm{tr}}$ is used in the transition zone between saturation and interpolation.

$$\boldsymbol{M}_{\mathrm{int}}(\boldsymbol{H}(t_i)) = \sum_{n=1}^{3} a_n(\boldsymbol{H}(t_i))\, \boldsymbol{M}_{\mathrm{N},n}(\boldsymbol{H}(t_i)) \tag{7}$$

The losses associated with the hysteretic behavior of ferromagnetic materials are calculated with a generalized version of the Barkhausen-jump-method, which was introduced in [6].

IV. Iteration Scheme

Despite its slow convergence rate compared to the Newton-Raphson-iteration a fixed-point iteration was used to solve the nonlinear problem due to its robustness and easy implementation. The iteration scheme will be presented in more detail in the full paper.

V. Simulation results

The computations where applied to the TEAM-Problem 32 (three limb transformer) and an exemplary model-problem for electric machines, which consists of a stator with a three-phase winding and a solid non-moving rotor. Selected calculation results of the last-mentioned are shown in Fig. 2. It becomes apparent, that the losses are concentrated in parts of the geometry which undergo a high alternating flux density, whereas areas exposed to a rather rotating flux density exhibit a lower density of dissipated energy.

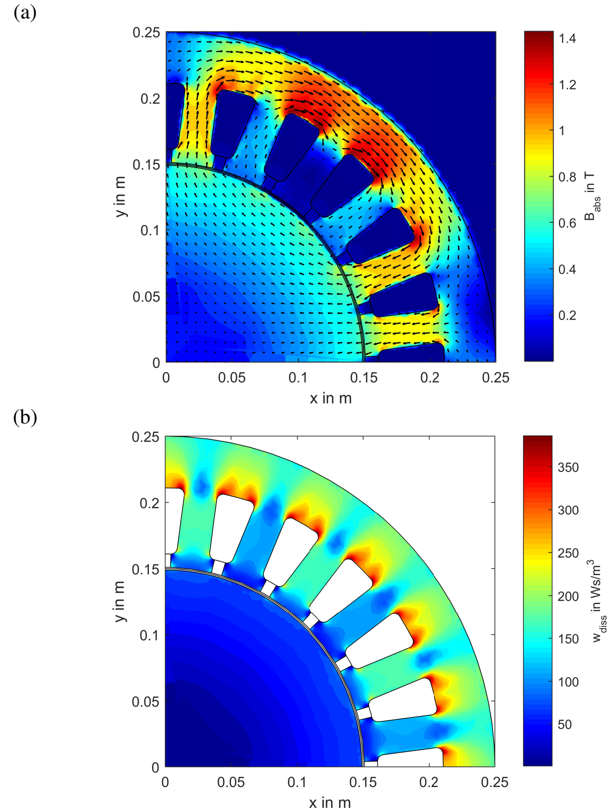

Fig. 2: Magnetic flux density (a) and corresponding density of dissipated energy (b) for the examined geometry

VI. Conclusion

The presented calculation scheme allows direct incorporation of hysteresis losses in the finite element analysis. The results are promising, however to be of greater practical use, the application of the Newton-Raphson-iteration is desirable. The combination of the hysteresis operator with an $\boldsymbol{A}$-based potential formulation is subject to ongoing research.

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